International Journal of Theoretical Physics, Vol. 16, No. 10 (1977), pp. 763–773

Diffraction of Long Waves by a Semi-Infinite Vertical Barrier on a Rotating Earth

GEORGE M. KAPOULITSAS

Dodekanisou 13, Heraklion, Crete, Greece

Received: 29 *September* 1977

Abstract

An asymptotic solution for the problem of diffraction of a plane wave by a rotating semi-infinite thin barrier is presented. Expressions are obtained for the wave amplitude and a particular discussion concerns the parts of the solution due to the rotational effect.

1. *Introduction*

In this paper the diffraction of long gravity water waves approaching a semi-infinite vertical barrier is discussed. The whole system is considered in rotation. Such problems arise from certain aspects of an investigation into the origin of storm surges.

The direction of the incident wave is arbitrary and it is shown that a wave due to rotation arises in the shadow region and travels along the barrier without attenuation. Crease (1956) has considered the case of an incident wave from a direction perpendicular to the barrier by using the Wiener–Hopf technique and constructing an appropriate integral equation. The method used here is also based on the Wiener–Hopf technique but in a simpler form and without the necessity of an integral equation formulation. The solution, when the rotation is taken to be zero, represents the usual diffraction effect in acoustics and electromagnetism [Sommerfeld's problem, cf. Copson (1946)].

2. *Formulation of the Problem*

The linearized equations of motion of a fluid sheet of constant depth h in the long-wave theory assuming a time factor $e^{-i\omega t}$, are, in Cartesian coordinates, the following (Proudman, 1953):

$$hk^2u_1 = -i\omega \frac{\partial \phi_t}{\partial x} + f\frac{\partial \phi_t}{\partial y} \tag{2.1a}$$

This journal is copyrighted by Plenum. Each article is available for \$7.50 from Plenum Publishing Corporation, 227 West 17th Street, New York, N.Y. 10011.

$$hk^2u_2 = -f\frac{\partial \phi_t}{\partial x} - i\omega\frac{\partial \phi_t}{\partial y} \tag{2.1b}$$

$$(\nabla^2 + k^2)\phi_t = 0 \tag{2.1c}$$

Here ϕ_t is the total elevation of the free surface above its mean level, (u_1, u_2) are the components of the velocity in the horizontal (x, y) plane, which are functions of x, y, t only, f is the Coriolis parameter equal to $2\Omega \sin \rho$, where Ω is the angular velocity of the earth and ρ the north latitude, and g the gravity acceleration. Also $k^2c^2 = \omega^2 - f^2$ with $\omega > f > 0$ and $c^2 = gh$.

Suppose that a plane wave

$$\phi_i = \exp(-ikx \cos \theta - iky \sin \theta), \qquad 0 < \theta < \pi \tag{2.2}$$

which satisfies equation (2.1c), is incident on a vertical rigid barrier of zero thickness along the negative x axis. We define the function ϕ by the equation

$$\phi_t = \phi + \phi_i \tag{2.3}$$

everywhere in the field. For convenience we can put

$$f = kc \sinh \beta, \qquad \omega = kc \cosh \beta$$

where β is real, since $\omega/kc > 1$. The problem is to find a solution ϕ of equation (2.1c) satisfying the following conditions:

$$\frac{\partial \phi}{\partial y} - i \tanh \beta \frac{\partial \phi}{\partial x} = ik(\sin \theta - i \tanh \beta \cdot \cos \theta) \exp(-ikx \cos \theta), \qquad y = 0 \pm 0, x < 0 \tag{2.4}$$

$$\phi(x, 0 + 0) = \phi(x, 0 - 0), \qquad x \geqslant 0 \tag{2.5}$$

and

$$\left(\frac{\partial \phi}{\partial y} - i \tanh \beta \cdot \frac{\partial \phi}{\phi x}\right)_{y = 0 - 0} = \left(\frac{\partial \phi}{\partial y} - \tanh \beta \cdot \frac{\partial \phi}{\partial x}\right)_{y = 0 + 0},$$

$$-\infty < x < \infty \tag{2.6}$$

Also we suppose that

$$\phi = O(1) \qquad \text{as } x \to 0 \pm 0, y = 0 \tag{2.7a}$$

and

$$\frac{\partial \phi}{\partial y} = O(x^{-1/2}) \text{ as } x \to 0 + 0, y = 0 \tag{2.7b}$$

In the following we assume that ω is complex with a small positive imaginary part ω_2 (i.e., $\omega = \omega_1 + i\omega_2$, $\omega_1 \gg \omega_2 > 0$) and this implies that k must also be complex with a small positive imaginary part (i.e., $k = k_1 + ik_2$, $k_1 \gg k_2 > 0$). This assumption is necessary for the application of the method of solution followed. The solution is obtained from the final results by taking $\omega_2 \to 0 + 0$, which implies $k_2 \to 0 + 0$.

It can be shown that for any $y \geqslant 0$

(a) $|\phi| = O[\exp(k_2 x \cos\theta - k_2 y \sin\theta)]$ for $-\infty < x < -y \cot\theta$

(b) $|\phi| = O(e^{-k_2|x|})$ for $-y \cot\theta < x < \infty$

and also for any $y \leqslant 0$

(c) $|\phi| = O(e^{\tau_0 x + y \sin\theta})$ for $-\infty < x < y \cot\theta$

where

$$\tau_0 = \min\left(\frac{\omega_2}{c}, k_2 \cos\theta\right) \text{ and } \frac{\omega_2}{c} = k_2\left(1 - \frac{f^2}{\omega_1{}^2 + k_2{}^2 c^2}\right) \leqslant k_2$$

and

(d) $|\phi| = O(e^{-k_2|x|})$ for $y \cot\theta < x < \infty$

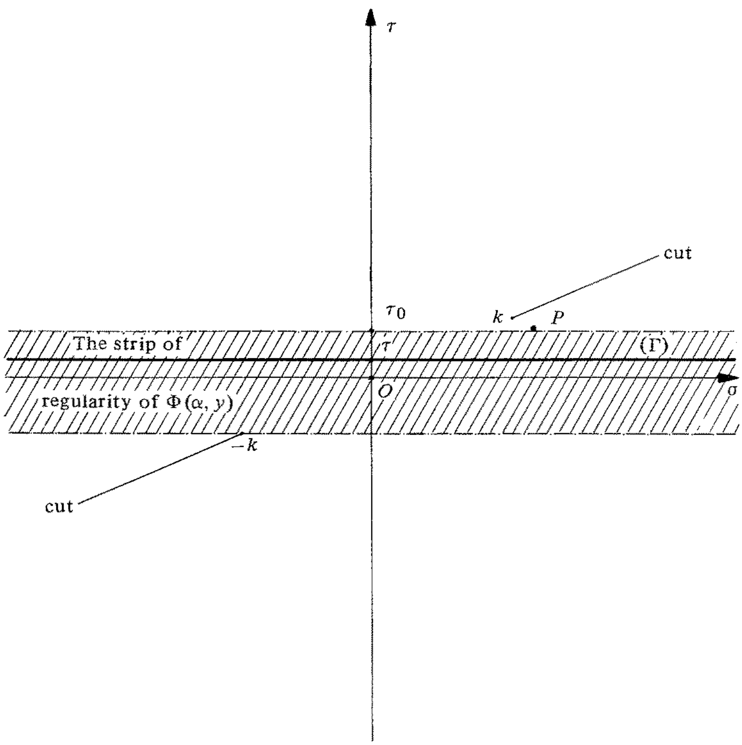

Figure 1. The path of integration Γ and the cuts in the complex α-plane.

Therefore the two-sided complex Fourier transform $\Phi(\alpha, y)$ of $\phi(x, y)$ in x, defined by

$$\Phi(\alpha, y) = \int_{-\infty}^{\infty} \phi(x, y)e^{i\alpha x}\, dx, \qquad \alpha = \sigma + i\tau (\sigma, \tau \text{ real}) \tag{2.8}$$

exists in the whole (x, y) plane and is regular in the strip $-k_2 < \tau < \tau_0$ (Figure 1). It is also found that $\Phi(\alpha, y)$ is bounded as $|y| \to \infty$ for any α in the above strip of regularity.

We introduce, further, the following one-sided complex Fourier transforms:

$$\Phi_+(\alpha, y) = \int_0^{\infty} \phi(x, y)e^{i\alpha x}\, dx \tag{2.9a}$$

and

$$\Phi_-(\alpha, y) = \int_{-\infty}^{0} \phi(x, y)e^{i\alpha x}\, dx \tag{2.9b}$$

which exist in the whole (x, y) plane and are regular in $\tau > -k_2$ and $\tau < \tau_0$, respectively.

3. *The Functional Equation*

We now proceed to derive an appropriate functional equation, called the Wiener–Hopf equation, of the problem.

Applying the Fourier transform to equation (2.1c), which also holds for ϕ, and taking into account its behavior as $|x| \to \infty$, assuming, further, that $\partial\phi/\partial x$ has similar behavior for $|x| \to \infty$, we get

$$\frac{d^2\Phi(\alpha, y)}{dy^2} - \gamma^2\Phi(\alpha, y) = 0, \qquad -k_2 < \tau < \tau_0 \tag{3.1}$$

where

$$\gamma = (\alpha^2 - k^2)^{1/2} \tag{3.2}$$

The function γ has branch points at $\alpha = \pm k$; the cuts from these points are taken symmetrical to each other with respect to the origin, outside the strip $|\tau| < k_2$ and along the straight line passing through these branch points. To obtain the physically acceptable solution of the problem, viz., the solution that satisfies all the requirements of Section 2 and further obeys Sommerfeld's "radiation condition" at infinity (1964), a suitable branch of the primarily multivalued function $(\alpha^2 - k^2)^{1/2}$ is specified by the author (Kapoulitsas, 1975).

We mention here that by this specification the real part of γ is positive inside the strip $|\tau| < k_2$.

The general solution of (3.2), regular in the strip $-k_2 < \tau < \tau_0$ and bounded for $|y| \to \infty$, is

$$\Phi(\alpha, y) = A(\alpha)e^{-\gamma y}, \qquad y \geqslant 0 \tag{3.3a}$$

$$= B(\alpha)e^{\gamma y}, \qquad y \leqslant 0 \tag{3.3b}$$

Moreover from (2.4)

$$\Phi'_-(\alpha, 0 \pm 0) - \alpha \tanh \beta \Phi_-(\alpha, 0 \pm 0) - i \tanh \beta \cdot \phi(0, 0)$$
$$= \frac{k(\sin \theta - i \tanh \beta \cdot \cos \theta)}{\alpha - k \cos \theta} = \frac{k \sin(\theta - i\beta)}{\cosh \beta} \cdot \frac{1}{\alpha - k \cos \theta} \tag{3.4}$$

provided that $\phi(0, 0+0) = \phi(0, 0-0) = \phi(0, 0)$. The primes on Φ's are taken to mean the derivatives with respect to y. Also from (2.6) we get

$$\Phi'(\alpha, 0-0) - \alpha \tanh \beta \cdot \Phi(\alpha, 0-0) = \Phi'(\alpha, 0+0) - \alpha \tanh \beta \cdot \Phi(\alpha, 0+0)$$

and using equation (3.3) we obtain

$$B = \frac{\gamma + \alpha \tanh \beta}{-\gamma + \alpha \tanh \beta} A \tag{3.5}$$

Next we define $\Psi_+(\alpha)$ as

$$\Psi_+(\alpha) \equiv \Phi'_+(\alpha, 0-0) - \alpha \tanh \beta \cdot \Phi_+(\alpha, 0-0) + i \tanh \beta \cdot \phi(0, 0)$$
$$= \Phi'_+(\alpha, 0+0) - \alpha \tanh \beta \cdot \Phi_+(\alpha, 0+0) + i \tanh \beta \cdot \phi(0, 0) \tag{3.6}$$

which is regular in the upper half-plane $\tau > -k_2$.

By virtue of (2.5), (3.3) and (3.4)

$$\Psi_+(\alpha) = -A(\gamma + \alpha \tanh \beta) - \frac{k \sin(\theta - i\beta)}{\cosh \beta(a - k \cos \theta)} \tag{3.7}$$

Now let us define

$$F_-(\alpha) \equiv \tfrac{1}{2}\{\Phi_-(\alpha, 0-0) - \Phi_-(\alpha, 0+0)\} \tag{3.8}$$

The function $F_-(\alpha)$ is regular in the lower half-plane $\tau > \tau_0$, and using (3.3) and (3.5), we have

$$F_-(\alpha) = \frac{\gamma}{-\gamma + \alpha \tanh \beta} \cdot A \tag{3.9}$$

Eliminating A between (3.7) and (3.9) we get finally

$$\Psi_+(\alpha) = \frac{\alpha^2 - k^2 \cosh^2 \beta}{\cosh^2 \beta} \cdot \frac{1}{\gamma} \cdot F_-(\alpha) - \frac{k \sin(\theta - i\beta)}{\cosh \beta(\alpha - k \cos \theta)} \tag{3.10}$$

Equation (3.10) is a functional equation of the Wiener-Hopf type.

4. *Solution of the Functional Equation*

Equation (3.10) contains two unknown functions $\Psi_+(\alpha)$ and $F_-(\alpha)$ which are regular in an upper and a lower half-plane, respectively, these half-planes having a common strip. To determine both these unknown functions we write the equation as

$$\Psi_+(\alpha)\frac{(\alpha+k)^{1/2}}{\alpha+k\cosh\beta}+\frac{k\sin(\theta-i\beta)}{\cosh\beta(\alpha-k\cos\theta)}\left[\frac{(\alpha+k)^{1/2}}{\alpha+k\cosh\beta}-\frac{(k\cos\theta+k)^{1/2}}{k(\cos\theta+\cosh\beta)}\right]$$

$$=F_-(\alpha)\frac{(\alpha-k\cos\beta)}{(\alpha-k)^{1/2}}-\frac{\sin(\theta-i\beta)}{\cosh\beta}\cdot\frac{(k\cos\theta+k)^{1/2}}{(\alpha-k\cos\theta)(\cos\theta+\cosh\beta)} \tag{4.1}$$

In equation (4.1) the left-hand side is regular in the lower half-plane $\tau < \tau_0$, while the right-hand side is regular in the upper half-plane $\tau > -k_2$. Therefore both sides are regular in the strip $-k_2 < \tau < \tau_0$ and, by analytic continuation, they define a function, $P(\alpha)$ say, which is regular over the entire α-plane.

To determine $P(\alpha)$ we have recourse to the asymptotic behavior of all the functions in (4.1).

Even though $\Psi_+(\alpha)$ is as yet unknown, its asymptotic behavior is governed by the edge conditions [equation (2.7)], since by applying the appropriate Abel theorem we find that, according to definition (3.6),

$$\Psi_+(\alpha) = O(1) \qquad \text{as } \alpha \to \infty \text{ with } \tau > -k_2 \tag{4.2}$$

Similarly from the edge condition (2.7a) and the definition (3.8) we find

$$F_-(\alpha) = O(\alpha^{-1}) \quad \text{as } \alpha \to \infty \quad \text{with } \tau < \tau_0 \tag{4.3}$$

Thus both sides of (4.1) are of order $|\alpha|^{-1/2}$ as $\alpha \to \infty$ in their appropriate half-planes. It then follows from an extension to Liouville's theorem on polynomials that $P(\alpha)$ is a polynomial of order less than $(-\frac{1}{2})$ and therefore a constant. This constant is zero because $P(\alpha) = O(|\alpha|^{-1/2})$ as $|\alpha| \to \infty$. Hence

$$\Psi_+(\alpha)=\frac{-k\sin(\theta-i\beta)(\alpha+k\cosh\beta)}{\cosh\beta(\alpha+k)^{1/2}(\alpha-k\cos\theta)}\left[\frac{(\alpha+k)^{1/2}}{\alpha+k\cosh\beta}-\frac{(k\cos\theta+k)^{1/2}}{k\cos\theta+k\cosh\beta}\right] \tag{4.4}$$

Therefore from (3.8)

$$A = E\frac{\gamma-\alpha\tanh\beta}{(\alpha+k)^{1/2}(\alpha-k\cosh\beta)(\alpha-k\cos\theta)} \tag{4.5a}$$

and from (3.5)

$$B = -E\frac{\gamma+\alpha\tanh\beta}{(\alpha+k)^{1/2}(\alpha-k\cosh\beta)(\alpha-k\cos\theta)} \tag{4.5b}$$

where

$$E=\frac{-\sin(\theta-i\beta)(k\cos\theta+k)^{1/2}\cosh\beta}{\cos\theta+\cosh\beta} \tag{4.6}$$

By virtue of equations (4.5) the Fourier transform of the solution of the problem, given by equation (3.3), is known.

5. *The Field Solution*

To find the field solution $\phi(x, y)$ from its Fourier transform we have to invert $\Phi(\alpha, y)$ using the formula

$$\phi(x, y) = \frac{1}{2\pi} \int_{-\infty+i\tau}^{\infty+i\tau} \Phi(\alpha, y) e^{-i\alpha x}\, d\alpha \tag{5.1}$$

The path Γ of the above integration is the straight line $\tau = \text{const}$, $-k_2 < \tau < \tau_0$ (Figure 1).

We shall determine $\phi(x, y)$ for $y \geqslant 0$ and $y \leqslant 0$ separately.

5.1 *Region A* ($y \geqslant 0$, $-\infty < x < \infty$). In this region we have, according to (3.3a) and (4.5a),

$$\Phi(\alpha, y) = E \frac{(\gamma - \alpha \tanh \beta) e^{-\gamma y}}{(\alpha + k)^{1/2} (\alpha - k \cosh \beta)(\alpha - k \cos \theta)} \tag{5.2}$$

The singularities of $\Phi(\alpha, y)$ are the simple pole at $\alpha = k \cos \theta$ and the branch points at $\alpha = \pm k$.

Because of the existence of the above branch points the integral (5.1) can be evaluated only asymptotically for large $r = (x^2 + y^2)^{1/2}$, by applying the method of steepest descent (Copson, 1970). Yet, we cannot apply this method directly to (5.2) since the pole at $\alpha = k \cos \theta$ may be near the saddle point. To avoid this difficulty we split $\Phi(\alpha, y)$ into two parts $\Phi_1(\alpha, y)$ and $\Phi_2(\alpha, y)$ so that

$$\Phi_1(\alpha, y) = E \left\{ \left[\frac{\gamma - \alpha \tanh \beta}{(\alpha + k)^{1/2} (\alpha - k \cos \theta)(\alpha - k \cos \theta)} \right] - a_{-1} \frac{(k \cos \theta - k)^{1/2}}{(\alpha - k)^{1/2} (\alpha - k \cos \theta)} \right\} e^{-\gamma y} \tag{5.3}$$

and

$$\Phi_2(\alpha, y) = E a_{-1} \frac{(k \cos \theta - k)^{1/2}}{(\alpha - k)^{1/2}} \cdot \frac{e^{-\gamma y}}{\alpha - k \cos \theta} \tag{5.4}$$

where a_{-1} is the residue of the term in the square brackets of (5.3) at the pole $\alpha = k \cos \theta$, which is found to be

$$\frac{-i \sin(\theta - i\beta)}{\cosh \beta (k \cos \theta + k)^{1/2} (\cos \theta - \cosh \beta)}.$$

Consider next the transformations

$$x = r \cos v, \qquad |y| = r \sin v, \qquad 0 < v < \pi \tag{5.5}$$

and

$$\alpha = -k \cos z, \qquad \gamma = -ik \sin z \tag{5.6}$$

where $z = p + iq$ (p, q real) which are applied to the integral in (5.1) and for the part $\Phi_1(\alpha, y)$, which has now no pole at all. The result is

$$\phi_1(x, y) = \frac{Ek}{2\pi} \int_{\Gamma'} \Phi_1(-k \cos z, y) \sin z \, e^{ikr\cos(z-v)} \, dz \tag{5.7}$$

From now on we take $k_2 \to 0+$ to avoid unnecessary complications. The transformed integration path Γ' consists of the segments $E'B'A'D'$ shown in Figure 2. The saddle point S is at $z_s = v$ (i.e., $p_s = v$, $q_s = 0$) and Γ' is deformed to the path LSM, since it can be shown that the contribution of the segments D'L and ME' into the integral of (5.7) is zero as $|q| \to \infty$.

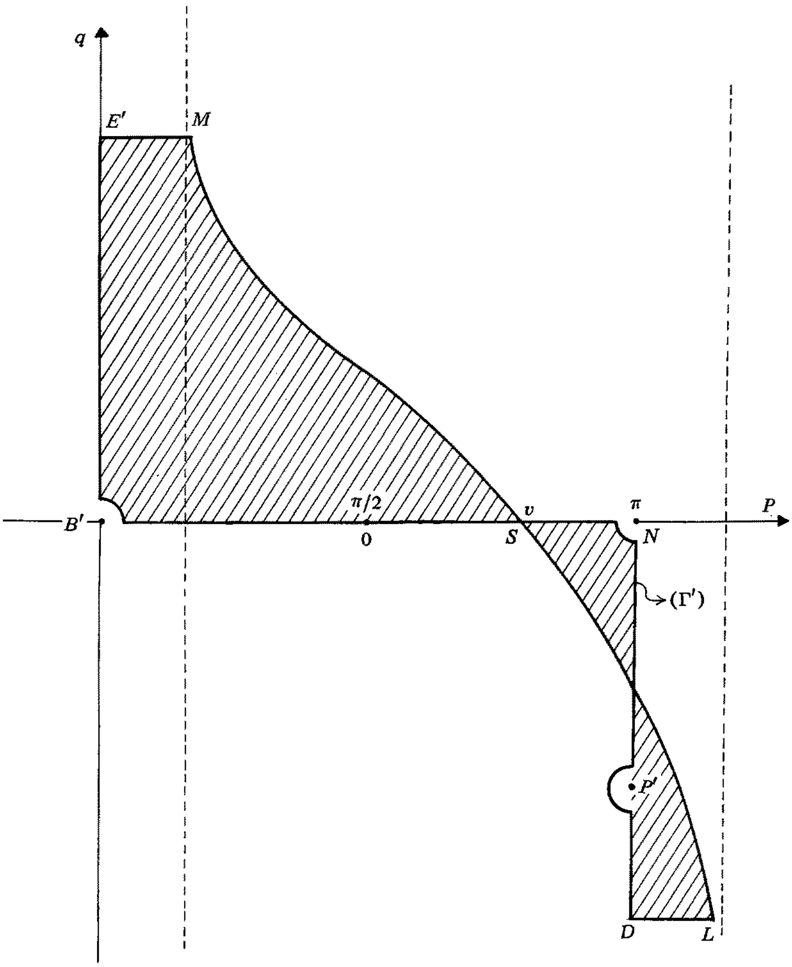

Figure 2. The path of integration Γ' and the path of steepest descent LSM in the complex z-plane.

Since the integrand in (5.7) has no pole at all, we, following the standard procedure of the method of steepest descent, obtain

$$\phi_1(x, y) = \frac{1}{2\pi} \cdot \frac{E}{(\cos v + \cos\theta)} \left[\frac{-i \sin(v + i\beta)}{k \cosh\beta(-k\cos v + k)^{1/2}(\cos v + \cosh\beta)} + a_{-1} \frac{(k\cos\theta - k)^{1/2}}{k(-k\cos v - k)^{1/2}} \right] \sin v \cdot \left(\frac{2k\pi}{r}\right)^{1/2} e^{i(kr - \pi/4)} \qquad (5.8)$$

The part $\phi_1(x, y)$ of the solution $\phi(x, y)$ in this region is entirely due to the rotational effect and represents a cylindrical Poincaré-type wave radiated from the origin with a phase velocity ω/k. This wave disappears for $f = 0$.

The contribution to the integral (5.1) from the second part $\Phi_2(\alpha, y)$ of $\Phi(\alpha, y)$ is

$$\phi_2(x, y) = \frac{\sin(\theta - i\beta)}{\sin(\theta + i\beta)} \left[-\frac{i}{2\pi}(k\cos\theta - k)^{1/2} \int_{-\infty + i\tau}^{\infty + i\tau} \frac{e^{-\gamma y - i\alpha x}}{(\alpha - k)^{1/2}(\alpha - k\cos\theta)} d\alpha \right]$$

$$= \frac{\sin(\theta - i\beta)}{\sin(\theta + i\beta)} \phi_0(x, y) \qquad (5.9)$$

where the term $\phi_0(x, y)$, equal to the quantity in the square brackets of (5.9), expresses the diffraction by the considered semi-infinite barrier when no rotation exists (Sommerfeld's problem) and has been fully discussed in the past (cf. Noble, 1958). Thus $\phi_2(x, y)$ apart from a phase change is the part of the solution that is not affected by the rotational effect.

5.2 *Region B* ($y \leqslant 0, -\infty < x < \infty$). From equations (3.3b) and (4.5b) we get

$$\Phi(\alpha, y) = \frac{-E(\gamma + \alpha\tanh\beta)}{(\alpha + k)^{1/2}(\alpha - k\cosh\beta)(\alpha - k\cos\theta)} e^{-\gamma|y|}$$

and following the same lines as in the case of region A we find that

$$\Phi(\alpha, y) = \Phi_3(\alpha, y) + \Phi_4(\alpha, y)$$

where

$$\Phi_3(\alpha, y) = E\left\{ \left[\frac{-\gamma - \alpha\tanh\beta}{(\alpha + k)^{1/2}(\alpha - k\cosh\beta)(\alpha - k\cos\theta)} \right] - b_{-1} \frac{(k\cos\theta - k)^{1/2}}{(\alpha - k)^{1/2}(\alpha - k\cos\theta)} \right\} e^{-\gamma|y|} \qquad (5.10)$$

and

$$\Phi_4(\alpha, y) = Eb_{-1} \frac{(k\cos\theta - k)^{1/2}}{(\alpha - k)^{1/2}(\alpha - k\cos\theta)} e^{-\gamma|y|} \qquad (5.11)$$

where b_{-1} is the residue of the term in the square brackets of (5.10) of the pole $\alpha = k\cos\theta$ and which is

$$\frac{i\sin(\theta + i\beta)}{\cosh\beta(k\cos\theta + k)^{1/2}(\cos\theta - \cosh\beta)}$$

$\Phi_3(\alpha, y)$ has no pole near the saddle point, but it has a pole P at $\alpha = k\cosh\beta$. Next, following the same lines of procedure as in the case of region A and taking into account that the above pole is transformed into P', Figure 2, which is captured by the shaded region only if $\pi > v > \cos^{-1}(kc/\omega)$, where $0 < \cos^{-1}(kc/\omega) < \pi/2$, we obtain for $\phi_3(x, y)$

$$\phi_3(x, y) = \phi_r + \phi_k \cdot H(v - v_0)$$

where

$$\phi_r = \frac{1}{2\pi} \cdot \frac{E}{\cos v + \cos\theta} \left[\frac{i\sin(v - i\beta)}{k\cosh\beta(-k\cos v + k)^{1/2}(\cos v + \cos\beta)} + b_{-1} \frac{(k\cos\theta - k)^{1/2}}{(-k\cos v - k)^{1/2}(\cos v + \cos\phi)} \right] \sin v \left(\frac{2k\pi}{r}\right)^{1/2} e^{i(kr - \pi/4)} \tag{5.12a}$$

$$\phi_k = 2i(\mathrm{Re}\, s)_{P_2'} = -2iE \frac{\sinh\beta \exp(-ix\cosh\beta + y\sinh\beta)}{(k\cosh\beta + k)^{1/2}(\cosh\beta - \cos\theta)} \tag{5.12b}$$

$H(v - v_0)$ is the Heaviside unit function and $v_0 = \pi - \cos^{-1}(kc/\omega)$. ϕ_r is a cylindrical wave (Poincaré wave) radiated from the origin and ϕ_k is a Kelvin wave (Proudman, 1953) traveling along the negative x axis. Both ϕ_r and ϕ_k are a consequence of the rotational effect and disappear when $f = 0$.

Lastly $\phi_4(x, y)$, corresponding to the inverse of $\Phi_4(\alpha, y)$, is found to be equal to $-\phi_0$ and represents the pure effect of diffraction of a nonrotating semi-infinite barrier.

In conclusion the solution of the problem for $y \leqslant 0$ is given by

$$\phi(x, y) = \phi_r + \phi_k \cdot H(v - v_0) - \phi_0 \tag{5.13}$$

6. *Discussion*

In the above procedure we have presupposed that f is positive, that is, in the sense of increasing v. If the rotation takes place in the opposite sense, f will be negative. Then the created Kelvin wave appears behind the barrier and all the above results can be extended to negative values of f, these results being referred to the frame of reference reflected in the line $y = 0$.

Regarding the Kelvin wave ϕ_k appearing in the region mentioned, which is in the front of the barrier (for $f > 0$), the barrier being in the left half of the x axis, we could note that this region is the only part of the field where traveling waves are not attenuated away from the edge of the barrier. If the barrier is in the right half of the x axis ($x > 0$), we should expect a Kelvin wave traveling down behind the barrier, and this leads to the remarkable conclusion that for a barrier of finite length, as an oblong island in the sea, there will be a circulation of progressive waves round the barrier in the clockwise direction. Thus a certain amount of energy will be trapped near the barrier as Crease has observed (1956).

In equation (5.13) the terms ϕ_3 and ϕ_0 are of order $O(r^{-1/2})$ for large r, while the term ϕ_k is of order $O(e^{-f/c|y|})$. Thus, when y is small as $r \to \infty$ (region near the barrier), the leading term in (5.13) is the Kelvin wave since it does not diminish with the distance in the x direction; when, for $r \to \infty$, also $y \to -\infty$, as happens for most values of v, ϕ_3 and ϕ_0 become dominant terms in (5.22) expressing asymptotically the surface elevation considered.

References

Copson, E. T. (1946). *Quarterly Journal of Mathematics,* **17**, 19.
Copson, E. T. (1970). *Asymptotic Expansions,* University Press, Cambridge.
Crease, J. (1956). *Journal of Fluid Mechanics,* **1**, 86.
Kapoulitsas, G. M. (1975). Ph.D. Thesis, University of Surrey.
Noble, B. (1958). *Methods based on the Wiener-Hopf technique,* Pergamon Press, New York.
Proudman, J. (1953). *Dynamical Oceanography,* Methuen.
Sommerfeld, A. (1964). *Partial Differential Equations in Physics,* Academic Press, New York.